Number Theory Round

30 minutes | 15 problems

1. A genie tells you to find three consecutive integers such that the middle number is divisible by 3, and the sum of all three numbers is divisible by 9. What is the first number of the first set of integers that satisfies his requirements?

2. Gandalf loves the number 6, as it equals the sum of its divisors, excluding itself. What is the next number after 6 that Gandalf loves?

3. In the land of Oz, Dorothy is arranging the letters in the word "LION." What's the probability that the vowels (I and O) will be next to each other?

4. Captain Hook has some amount of pirate crew members. The number is a three-digit number divisible by both 18 and 51. The tens digit is smaller than the ones digit, and the sum of the tens and ones digit is smaller than the hundreds digit. How many crew members does Captain Hook have?

5. Merlin has a list of ingredients labeled a, b, c, d, e, f, g, and h. What is the probability that he picks up ingredients a, c, and h out of 4 randomly chosen ingredients without replacement?

6. An evil curse randomly falls upon innocent Muggles in the Muggle world every year. Out of all Muggles cursed, there are always 2.5% of them with the last name Magic. If 925 Muggles with the last name Magic were evilly cursed in 2023, how many Muggles were cursed in total that year?

7. When you stumble into Shrek's swamp, he gets really mad and threatens to turn you into a green stew unless you can help him solve a hard math problem that he can't figure out. Shrek wants you to find the sum of all the digits $4^5 \times 5^{10}$. What do you respond with to save yourself?

8. A bad wizard is trying to cast spell A, but sometimes casts spell B and spell C instead. The probability that he casts spell B is 30%, and the probability he casts spell C is the same as if two fair 10-sided dice both land on either the number 1 or 3. What is the probability he casts spell A when he tries to cast it, as a percentage?

9. Two spinners with sections 1, 2, 3, and 4 are spun with an equal probability of the pointer landing on each number. What is the probability that the sum of the two selected numbers is even?

10. The Mad Hatter tried to add all the integers from 1 to 11. However, he forgot to add a number. The resulting sum was a perfect cube. What number did he leave out?

11. While walking through Wonderland, you were stopped by the Cheshire Cat. The Cheshire Cat told you that:

$$a \twoheadrightarrow b = a^2 - b^2$$
$$a \hat{\oplus} b = (a + b)(a^2 - ab + b^2)$$

Evaluate $(5 \twoheadrightarrow (6 \hat{\oplus} 7))$ to pass and continue on your merry way.

12. Tim uncasts his magical spell that rigs dice. When 3 fair 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 16 can be written as $\frac{1}{n}$, where n is a positive integer. What is n?

13. Dumbledore, uniformly and at random, chooses a real number of pieces of magical sparkles to fix his wand from the interval [0, 1012]. Independently, Harry Potter chooses a real number of magical sparkles uniformly at random from the interval [0, 2024] to fix his broom. What is the probability that Dumbledore's number is smaller than Harry Potter's?

14. In the Magical world, the probability that a team wearing a red scarf and blue broom will win a game of Quidditch is the same as if 3 dice are rolled and their sum is not divisible by 7. In simplest fraction form, what is the chance that Durmstrang wins against Hogwarts if Durmstrang wears red scarves and blue brooms?

15. Two wizards caught sneaking into the Hogwarts library are given an integer ID. Their IDs are inserted into the list 26, 26, 58, 43, 74, doubling its range. The mode and median remain unchanged. What is the maximum possible sum of the two IDs?